MATH 140 Practice Final

1. What is the domain of the function defined by $f(x) = \ln(\ln x)$? (Answer: all $x > 1$)

2. If $h(x) = 3 + x^3 + e^{2x}$, what are the values of $h^{-1}(4)$ and $(h^{-1})'(4)$? (Answer: $h^{-1}(4) = 0$ and $(h^{-1})'(4) = \frac{1}{2}$)

3. What is the equation of the line tangent to the ellipse $3x^2 + 4y^2 = 12$ at the point $(-1, \frac{3}{2})$? (Answer: $y = \frac{1}{2}x + 2$)

4. If h is very small, what is the best approximation to the value of $\sin^{-1}(\frac{1}{2} + h)$? (Hint: Linearization; Answer: $\frac{\pi}{6} + \frac{2}{\sqrt{3}}h$)

5. What are the global extrema of the function $f(x) = x^4 - 4x^2 + 5$ on the interval $[-2, 2]$? (Answer: global minimum is 1 and global maximum is 5)

6. What is $\lim\limits_{x \to 0} \dfrac{2^x - \cos(x^2)}{x}$? (Hint: L'Hopital; Answer: $\ln 2$)

7. The velocity of a car chased by the police is given by $v(t) = t^2 e^{-t}$ ft/sec, where t is the time in seconds. What is the average velocity of the car during the first 2 seconds of this chase? (Hint: Integration by parts; Answer: $1 - 5e^{-2}$ ft/sec)

8. If $f(x) = (\sin x)^x$, what is $\dfrac{df}{dx}\left(\dfrac{\pi}{2}\right)$? (Hint: Logarithmic differentiation; Answer: 0)

9. If f is a continuous function such that

$$x^2 \sin(\pi x) = \int_1^{x^2} f(\sqrt{t})\, dt$$

for all x, what is $f(x)$? (Hint: Take derivative of both sides with respect to x; Answer: $f(x) = \sin(\pi x) + \frac{\pi}{2} x \cos(\pi x)$)

10. What is $\displaystyle\int \dfrac{2x^2 - x + 9}{x^3 - x^2 + 4x - 4}\, dx$? (Hint: Partial fractions; Answer: $2\ln|x - 1| - \frac{1}{2}\tan^{-1}(\frac{x}{2}) + C$)

11. What is $\displaystyle\int \sin\sqrt{x}\, dx$? (Hint: First a substitution, then integration by parts; Answer: $-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$)

12. What is the numerical value of $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sin\theta}{\cos^2\theta + 4}\, d\theta$? (Hint: Substitution; Answer: $\frac{1}{2}\tan^{-1}(\frac{1}{2})$)

13. What is the numerical value of $\int_{-\frac{1}{3}}^{\frac{1}{3}} \sqrt{1 - 9u^2}\, du$? (Hint: Use a trigonometric substitution; Answer: $\frac{\pi}{6}$)

14. A radioactive compound has half-life of T hours. An experiment starts with 16 grams of this compound; after 2 days only 0.25 grams of the sample is left. What is T? (Answer: $T = 8$ hours)

15. Car A is leaving an intersection at 20 mph heading north. Car B is approaching this intersection at 30 mph heading west. How fast is the distance between them changing at the moment when A is 3 miles north and B is 4 miles east of the intersection? (Answer: At that moment, the distance is decreasing at the rate of 12 mph)