Math 143 Second Midterm Review Sheet

11/3/2017

Here is an itemized list of the material that the second midterm is based upon. Make sure that you study them carefully. For each topic, you can review the examples that I gave in lecture, the worked out examples in the book, past quizzes, and the homework problems and solutions available on WebAssign.

- Improper integrals
- Sequences and their limits, basic limit laws for sequences, the sandwich lemma (aka squeeze theorem)
- Increasing and decreasing sequences, bounded monotonic sequences are convergent
- Infinite series and the meaning of their convergence
- The geometric series:

$$\sum_{n=k}^{\infty} r^n \begin{cases} \text{converges to } \frac{r^k}{1-r} & \text{if } |r| < 1 \\ \text{diverges} & \text{if } |r| \geq 1. \end{cases}$$

- The p-series:

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges} & \text{if } p > 1 \\ \text{diverges} & \text{if } 0 < p \leq 1. \end{cases}$$

  In particular, the harmonic series $\sum_{n=1}^{\infty} 1/n$ diverges while the Euler series $\sum_{n=1}^{\infty} 1/n^2$ converges.

- Basic divergence test: If $\{a_n\}$ doesn't tend to zero, the series $\sum_{n=1}^{\infty} a_n$ diverges.
- The integral, comparison, and limit comparison tests for series with positive terms.

Practice Problems

1. Show that if $p > 1$, the improper integral

$$\int_2^{\infty} \frac{dx}{x(\ln x)^p}$$

converges and find its value.

2. Find $\lim_{n\to\infty} a_n$ in each of the following cases:

- $a_n = \sin(1/n)$
- $a_n = (-1)^n \sin(1/n)$
- $a_n = n\ \sin(1/n)$
- $a_n = (-1)^n\ n\ \sin(1/n)$

3. Determine the convergence/divergence of the following series. In each case, specify the test you're using:

- $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$
- $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$
- $\sum_{n=1}^{\infty} \frac{\sqrt{n}+2}{n+1}$
- $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

4. Find the value of x such that

$$\sum_{n=1}^{\infty} 2^{-n} x^n = 5.$$

(Hint: Use the formula for the sum of a geometric series.)