MAT 160, PROBLEM SEMINAR, WEEK OF 2/1/99

PROBLEM SET 2: METHOD OF MATHEMATICAL INDUCTION

Problem 8. Prove by induction:

$$1 + 3 + 5 + \ldots + (2n-1) = n^2 \text{ for all integers } n \geq 1$$

(*Remark*: After proving this statement by induction, see if you can discover a "geometric" way to prove this, say by considering an $n \times n$ square.)

Problem 9. Prove that for all integers $n \geq 1$, the number $n^3 + (n+1)^3 + (n+2)^3$ is a multiple of 9.

Problem 10. Prove that for all integers $n \geq 5$, we have $2^n > n^2$.

Problem 11. Guess a formula for the sum $1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1)$ and prove it by induction.

Problem 12. "All horses are the same color." We can prove this by induction on the number of horses in a given set. Here is how: "Let $P(n)$ be the statement that given any set of n horses, they all are the same color. $P(1)$ is trivially true since if there is just one horse, then it is the same color as itself. Assume $P(n)$ (the induction hypothesis). To prove $P(n+1)$ (the induction step), take any set of $n+1$ horses. By the induction hypothesis, horses 1 through n are the same color, and similarly, horses 2 through $n+1$ are the same color. So horses 1 through $n+1$ must be the same color as well. This proves $P(n+1)$." What, if anything, is wrong with this reasoning?

Problem 13. Let $P(n)$ be the statement that a person may live n seconds. We prove $P(n)$ for all integers $n \geq 1$ by induction on n: "$P(1)$ is trivially true; all of us have lived 1 second. Assuming $P(n)$, it is easy to see that $P(n+1)$ must be true: If we assume that a person may live n seconds, then it is a safe bet to say that a person may also live only second more, i.e. $n+1$ seconds." Is something wrong with this proof? Note that if correct, the statement implies that you may find people as old as you want!

Problem 14. The plane is cut into regions by a finite number of straight lines. Prove that one can color the regions either (R)ed or (B)lue so that any two adjacent regions have different colors. Here adjacent means that they share a common boundary segment. (*Hint*: Use induction on the number of lines.)