# Math 360 Homework 1

*If you know how to read* **and** *write, you are literate. If you know how to read* **or** *write, you are a specialist.*

**Problem 1.** Let the universe of discourse be the set of all human beings. Let $P(x)$ be "$x$ is educated," $Q(x)$ be "$x$ is female," and $R(x)$ be "$x$ is older than 30." Thus, for example, the statement "Every uneducated male is older than 30" can be written as

$$\forall x : (\sim P(x) \wedge \sim Q(x)) \Rightarrow R(x)$$

Express the following statements in a similar fashion:

(a) Some educated people are younger than 30.
(b) Every female who is older than 30 is educated.
(c) No uneducated person is both female and older than 30. (*Hint:* It would be easier to think of the equivalent statement that every uneducated person is either male or younger than 30.)

**Problem 2.** Negate the following statement: "If your glass is half-empty, you are a pessimist or you are thirsty."

**Problem 3.** Let $A$, $B$, and $C$ be arbitrary sets. Show that

$$A \cap B \subset A \subset A \cup C$$

**Problem 4.** Let $S$ consist of the 26 letters of the alphabet. Let $A$ consist of all the consonants (including $y$), and $B$ be the letters that occur in *real functions* ($n$ being counted once). Show that $A \cup B = S$ and $A^c \subset B$.

**Problem 5.** Let $A$ and $B$ be arbitrary sets. Show that the sets $A \setminus B$ and $B \setminus A$ are disjoint.

**Problem 6.** Under what condition do we have $A \setminus (A \setminus B) = B$? Guess the answer using a diagram and then prove it carefully.

**Problem 7.** For $n = 1, 2, 3, \ldots$, let $A_n$ denote the interval $[\frac{1}{n}, +\infty[$ on the real line. Find

$$\bigcup_{n=1}^{\infty} A_n \quad \text{and} \quad \bigcap_{n=1}^{\infty} A_n$$