MATH 360 Homework 1

If you know how to read **and** *write, you are literate. If you know how to read* **or** *write, you are a specialist.*

Problem 1. Let the universe of discourse be the set of all human beings. Let $P(x)$ be "x is educated," $Q(x)$ be "x is female," and $R(x)$ be "x is older than 30 years of age." Thus, for example, the statement that "Every uneducated male is older than 30" can be written as

$$(\forall x)((\sim P(x) \wedge \sim Q(x)) \Rightarrow R(x))$$

Now express the following statements in a similar fashion using quantifiers:

(a) Some educated people are younger that 30.
(b) Every female who is older than 30 is educated.
(b) No uneducated person is both female and older that 30. (Hint: It is easier to think of the equivalent statement that every uneducated person is either male or younger than 30.)

Problem 2. Smarty claims that there is a positive real number x such that $x < \frac{1}{n}$ for all natural numbers n. If you were to disprove this, how would you formulate your argument? (You don't have to prove anything here; just state in English what you would prove in order to show that Smarty is wrong.)

Problem 3. Let A, B, and C be arbitrary sets. Prove that

$$A \cap B \subset A \subset A \cup C$$

Problem 4. Let A and B be arbitrary sets. Show that the sets $A \backslash B$ and $B \backslash A$ are disjoint.

Problem 5. Let S consist of the 26 letters of the alphabet. Let A consist of all the consonants (including y), and B of the letters that occur in the word *real functions* (n being counted once). Show that (a) $A \cup B = S$, (b) $A^c \subset B$, (c) $B^c \subset A$, (d) $A \cap B$ and A^c are disjoint.

Problem 6. Under what condition do we have $A \backslash (A \backslash B) = B$? Guess the answer using a diagram and then prove it carefully.

Problem 7. Let $A_1 \subset A_2 \subset \cdots \subset A_n$. What are the two sets

$$A_1 \cap A_2 \cap \cdots \cap A_n \qquad \text{and} \qquad A_1 \cup A_2 \cup \cdots \cup A_n?$$