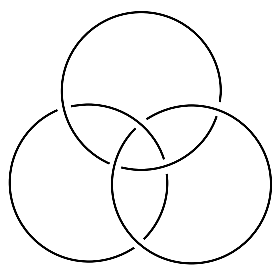
Math 364 Homework 7 (due 11/1/2002)

Problem 1. Draw a diagram for the connected sum of

(i) two copies of the right trefoil knot;
(ii) a right and a left trefoil knot;
(iii) two copies of the figure eight knot.

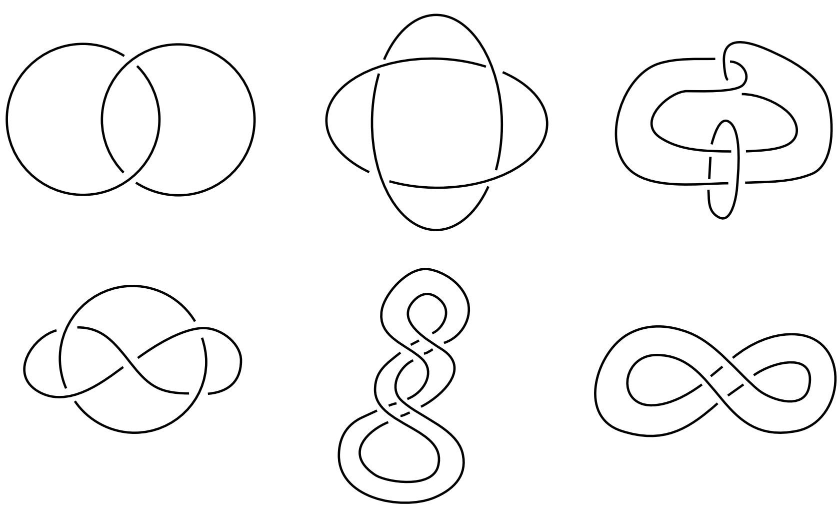
Problem 2. Suppose K_1 and K_2 are colorable knots. Show that the connected sum $K_1 \# K_2$ is also colorable.

Problem 3. In the following 3-component link, show that one of the components can be separated from the other two:

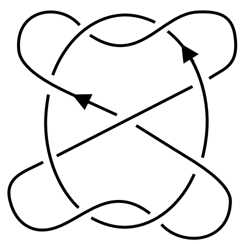
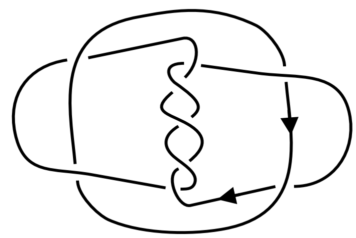


(Caution: This diagram does *not* represent the Borromean rings. If you look carefully, you see the difference.)

Problem 4. Each link in the first row is equivalent to a link in the second row (the ones in the first row, from left to right, are the Hopf link, King Solomon's knot, and the Whitehead link). Match the equivalent pairs and show by simple drawings how you deform one to another.



Problem 5. Compute the linking number of the oriented 2-component links represented by the following diagrams:

Problem 6. For each integer n, find an oriented 2-component link with linking number n. (Hint: The case $n = 0$ is easy and the case $n < 0$ can be reduced to $n > 0$ by reversing the orientation of one of the components. For $n > 0$, try a few small values to get the idea for the general case.)

Problem 7. (Optional Bonus Problem) This exercise demonstrates that the linking number $L(K, J)$ of an oriented 2-component link with components K and J is always an integer. Recall that

$$L(K, J) = \frac{1}{2} \sum_{p \in C} I_p$$

where C is the set of all crossings of K and J and the index I_p is $+1$ or -1 depending on whether p is a right or left crossing. The goal is to show that the sum is always an even integer.

Let $C_J \subset C$ be the set of crossings where J passes over K. The idea is to "switch" crossings in C_J one at a time and monitor what happens to the sum (by "switching" we mean changing over to under and vice versa).

(i) Show that if one crossing in C_J is switched, the sum $\sum_{p \in C} I_p$ either increases or decreases by 2.

(ii) Show that if all crossings in C_J are switched, the sum $\sum_{p \in C} I_p$ increases or decreases by an even integer.

(iii) In the diagram obtained in (ii) K always passes over J. It follows that the components K and J in this diagram can be separated and so their linking number is zero. Conclude that the original sum must have been an even integer.